

Stochastic multivariate analysis of hydrometeorological variables for wind erosion models

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Introduction

The stochastic modeling of wind speed has often concentrated either on multivariate models of daily wind speed, temperature, and solar radiation (e.g., Parlange and Katz, 2000), or on the simulation of hourly wind with no account for the mutual dependence on other hydro-climatic variables (e.g., Skidmore and Tatarko, 1990). The study of aeolian erosion requires a simultaneous modeling of high-resolution (e.g., hourly) wind speed, as well as of other weather variables (such as solar radiation, precipitation, air and dew-point temperature) needed in the analysis of surface soil moisture and of soil susceptibility to wind erosion. A multivariate stochastic process is here suggested as a possible approach to the multivariate modeling of these variables at the hourly time scale. The model is fitted to the data from two locations across the High Plains of the U.S., a region particularly affected by wind-blown dusts. The performances of the model are finally tested against the data records.

Methods

The present study uses Richardson's approach (Richardson, 1981; Parlange and Katz, 2000) to develop a multivariate model of wind components, air and dew-point temperatures at the hourly time scale. This model accounts for the diurnal variability, the autocorrelation, and the lagged cross-correlation of these variables, as well as for the inhomogeneity between rainy and dry days. The annual cycle is divided into 12 one-month-long segments; each of them is assumed to be unaffected by non-stationarities due to the seasonal cycle and is separately modeled. For each variable, the ensemble mean and standard deviation of the daily cycle are calculated month by month. Non-stationarity due to the daily cycle is removed from the time series by subtracting the mean daily cycle from the time series and dividing by the time-dependent standard deviation

$$X_i' = \frac{(X_i(t) - \langle X_i \rangle_t)}{\sigma_i(t)} \quad (i=1, \dots, 4) \quad (1)$$

where the subscript i indicates the hydroclimatic variable under question ($X_1=W_x$; $X_2=W_y$; $X_3=T_d$; $X_4=T$), while $\langle X_i \rangle$ and $\sigma_i(t)$ represent the daily cycle of the mean and standard deviation, respectively. The standardized variables, X_i' , are then normalized using a Box-Cox transformation (e.g. Hipel and McLeod, 1994; p. 122)

$$X_i''(t) = \begin{cases} X_i'^{\lambda_i} & \lambda_i \neq 0 \\ \ln(X_i') & \lambda_i = 0 \end{cases} \quad (i=1,\dots,4) \quad (2)$$

where values of the exponent λ_i are selected that are able to give variables X_i'' with distributions closest to normal. We show that the values of these standardized and normalized hydroclimatic variables are affected by precipitation occurrences, due to the impact that rainstorms have on air temperature, atmospheric humidity, and winds. This dependence has been assessed by separately estimating the means, $\mu_{i,j}$, and standard deviations, $\sigma_{i,j}$, of X_i'' for non-rainy or *dry* ($j=0$), and rainy or *wet* ($j=1$) days. These means and variances are found to be significantly different (95% confidence limit) in the t-test and F-test, respectively, supporting the hypothesis that these hydroclimatic variables need to be modeled conditionally upon the occurrence of precipitation. This inhomogeneity has been removed by standardizing the variables X_i'' on the condition that the day is dry or wet

$$X_i'''(t) = \frac{X_i''(t) - \mu_{i,j}}{\sigma_{i,j}} \quad (i, j = 1, \dots, 4) \quad (3)$$

with j being 1 or 0 depending on the (hourly) occurrence or non-occurrence of precipitation, respectively.

Rainfall occurrence has been modeled at the hourly time scale as a first-order two-state Markov chain (e.g., Gabriel and Neumann, 1962), where these states represent the occurrence ($j=1$) and non-occurrence ($j=0$) of rain. The only parameters of this rainfall occurrence process is represented by the transition probabilities P_{jk} ($j, k=0, 1$), indicating the probability that at time t the system will be in state k , with j being the state at time $t-1$. A likelihood ratio test for the order of the Markov chain (e. g. Wilks, 1995; p. 301) shows that the values of both the AIC (Akaiake's Information Criterion) and BIC (Bayesian Information Criterion) statistics are very close in the cases of first- and second-order Markov chain. A first-order process is here chosen to model rainfall occurrences, discarding for simplicity the idea of using different orders for the different months of the year. The hourly precipitation depth in the wet hours is modeled as a random variable with gamma distribution. No account is taken for the dependence of rainfall likelihood on the time of the day, since it was not found to be significant in the data.

The hydroclimatic time series in question are modeled using a multivariate second-order autoregressive process AR(2) (e.g. Hipel and McLeod, 1994) fitted to the standardized, normalized, and homogenized variables, X_i''' (equation (3)). The choice of a multivariate model is motivated by the existence of a significant ($p < 0.05$) cross-correlation between most of the variables in question. The choice of a second-order autoregressive process (AR(2)) is suggested by the inability of the AR(1) model to provide a good representation of the autocorrelation functions. Previous work on the (univariate) stochastic modeling of hourly wind speed has resulted in the use of AR(2) processes (Daniel and Chen, 1991; Brown et al., 1984; Nfaoui et al., 1996). All such studies remark the complexity existing in the modeling of the autocorrelation structure of these hourly variables, due to the presence of some residuals of the daily cycle, as well as

of seasonal nonstationarity observable in these one-month- long segments of the annual cycle. This study differs from these previous analyses in the use of a multivariate approach in the modeling of hourly wind speed components, air and dew point temperatures conditionally upon the occurrence of precipitation. We believe that the mutual dependence between these variables significantly affects the estimates of soil loss by wind erosion.

A problem is found with the modeling of the dew-point temperature, due to the physical constraint $T_d < T$. Even though this condition is partly accounted for by the mean, standard deviation and cross-correlation values of T and T_d , there are still chances to generate pairs of values of T and T_d with no physical meaning (i.e. with $T < T_d$). This is resolved by adjusting the values of dew-point temperature, imposing an upper bound on T_d . We show that this truncation does not significantly affect the statistics of T_d .

Solar radiation is the other variable needed in the modeling of the energy and moisture fluxes at the soil-atmosphere interface. The values of solar radiation at the ground surface depend on the sky clearness as well as on the extraterrestrial solar radiation. The latter is a deterministic function of latitude and time of the year, while the former depends on atmospheric conditions (i.e. clouds, water vapor, and aerosols). Therefore a number of stochastic weather generators (e.g., Hansen, 1999; Hensen and Mavromatis, 2001) model the sky clearness instead of surface solar radiation. Sky clearness, which is defined as the ratio between surface and extraterrestrial solar radiation, captures the random component of solar radiation. The lack of a dense network of direct measurements of solar radiation suggested (e.g., Lindsey and Farnsworth, 1997) using indirect measurements of sky clearness; the sky cover observations, S_c , are often adopted as proxies for sky clearness. S_c is a function of water vapor, clouds and aerosols. Therefore, sky cover is modeled as a random variable conditionally on rainfall occurrence, and on the quartiles of relative humidity (which is a function of dew point temperature).

Results

The model described in the previous sections has been used to simulate wind speed components, dew point temperatures, and precipitation. The results of these simulations have been compared with the data to assess to what extent the model is able to fit the observations. The two case studies analyzed in this work refer to the cities of Dodge City (KS) and Lubbock (TX) in the years 1961-1990. Table I reports month by month both the observed (Dodge City) and the simulated values of mean and standard deviation, showing a good agreement between data (NREL, 1992) and model results.

Table I. Simulated and observed (in parentheses) values of monthly means and standard deviations of the hydro-meteorological variables for Dodge City (KS).

W_x	<u>Mean</u> ($m\ s^{-1}$)	<u>Standard</u> <u>Deviation</u> ($m\ s^{-1}$)	T_{dew}	<u>Mean</u> ($m\ s^{-1}$)	<u>Standard</u> <u>Deviation</u> ($m\ s^{-1}$)
Mar	0.095 (0.117)	6.431 (6.320)	Mar	-2.674 (-2.210)	5.867 (5.780)
Jun	-2.583 (- 2.650)	4.964 (5.064)	Jun	13.620 (13.861)	3.796 (3.991)
Sep	-1.758 (- 2.030)	5.311 (5.322)	Sep	11.318 (11.149)	4.954 (5.289)
Dec	0.476 (0.454)	5.476 (5.513)	Dec	-6.706 (-6.691)	6.194 (6.111)
W_y	<u>Mean</u> ($m\ s^{-1}$)	<u>Standard</u> <u>Deviation</u> ($m\ s^{-1}$)	T	<u>Mean</u> ($m\ s^{-1}$)	<u>Standard</u> <u>Deviation</u> ($m\ s^{-1}$)
Mar	-0.662 (- 0.304)	3.801 (3.890)	Mar	6.138 (6.130)	7.949 (7.800)
Jun	0.488 (0.539)	3.087 (3.087)	Jun	23.703 (23.465)	6.285 (6.165)
Sep	0.555 (0.430)	2.714 (2.796)	Sep	20.098 (20.165)	6.838 (7.023)
Dec	-1.041 (- 1.222)	3.005 (3.040)	Dec	-0.352 (-0.258)	7.261 (7.448)

Moreover, as expected, the model is able to provide a good representation of the mutual dependence existing between these variables at lags of 0, 1, and 2 hours. The model's ability to reproduce the autocorrelation structure of these time series has been evaluated through the analysis of the residual autocorrelation function. In most of the months the hypothesis that the residuals are uncorrelated is true for wind components and dew point temperature, while it is not verified for the air temperature, due to the existence of some residues of the daily cycle as well as to possible non-stationarity existing in the one-month-long segments of the annual cycle. Nevertheless, even though more refined filtering techniques are being considered to remove such a periodicity, this model gives an overall realistic representation of the climatological variables needed in the modeling of wind erosion.

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